Bachelor of Science (B.Sc.) Semester—II (C.B.S.) Examination

MATHEMATICS

(Vector Calculus and Improper Integrals)

Compulsory Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) A particle moves so that its position vector is given by $\bar{r} = \cos wt\,\bar{i} + \sin wt\,\bar{j}$, where w is constant. Show that :

(i) the velocity $\bar{v}$ of the particle is perpendicular to $\bar{r}$.

(ii) the acceleration $\bar{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin.

(iii) $\bar{r} \times \bar{v}$ = a constant vector. 6

(B) Prove that $r^n.\bar{r}$ is irrotational. Find the value of n when it is solenoidal. 6

OR

(C) If $\bar{v} = \bar{w} \times \bar{r}$, where = $\bar{w} = w_1\bar{i} + w_2\bar{j} + w_3\bar{k}$ is a constant vector, $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then prove that

$$\bar{w} = \frac{1}{2}\text{ curl } \bar{v}.$$ 6

(D) Find the workdone in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). 6

UNIT—II

2. (A) Find the area lying between the line $y = x$ and the parabola $y^2 = 4x$. 6

(B) By changing the order of integration, evaluate :

$$\int_0^2 \int_0^x (x + y^2)\,dy\,dx .$$ 6

OR

(C) Evaluate $\int_0^2 \int_0^{\sqrt{1-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$ by changing to polar coordinates. 6

(D) Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 dz\,dy\,dx$. 6

UNIT—III

3. (A) Evaluate $\iiint_v x^2 y\,dv$, where v is the closed region bounded by the planes $x + y + z = 1$, $x = 0$, $y = 0$, $z = 0$. 6

(B) State Green's theorem. Using Green's theorem, evaluate the line integral $\oint_C (xy + y^2)dx + x^2 dy$, where C is the closed curve bounded by $x = \sqrt{y}$ and $y = x$. 6

OR

(C) Verify Stoke's theorem for $\vec{A} = (y - z + 2)i + (yz + 4)j - xzk$, where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane. 6

(D) Verify Divergence theorem for $\bar{A} = 2x^2 y\bar{i} - y^2\bar{j} + 4xz^2\bar{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$. 6

UNIT—IV

4. (A) Test the convergence of

(i) $\int_2^{\infty} \frac{x^2 - 1}{\sqrt{x^6 + 16}}\,dx$

(ii) $\int_1^{\infty} \frac{1 - \cos x}{x^2}\,dx$. 6

(B) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 6

OR

(C) Prove that $\beta(m, n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$,

Hence show that

$$\int_0^{\frac{\pi}{2}} \sin^p\theta \, \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}.$$ 6

(D) Evaluate using β–γ function :

(i) $\int_0^{\frac{\pi}{2}} \cos^6\theta \sin^2\theta \, d\theta$.

(ii) $\int_0^{\infty} \frac{x^{-3/2}}{1+x} dx$. 6

Question—V

5. (A) Find $\overline{\nabla}\phi$ at the point (1, 2, 0) if $\phi = x^2y + 2xz$. 1½

(B) Evaluate $\int_c F \cdot dr$, where $\vec{F} = 2xi + yj$, and C is the straight line y = x from (0, 0) to (1, 1). 1½

(C) Evaluate $\int_0^1 \int_0^2 (x + y^2) \, dx \, dy$. 1½

(D) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^2 \gamma^2 \, dr \, d\theta$. 1½

(E) Apply Green's theorem to show that the area enclosed by a simple plane curve C is $\frac{1}{2}\oint_{c}(x\,dy - y\,dx)$. 1½

(F) Evaluate $\iint_{s} \vec{r}\,.\,\vec{n}\ dS$, where S is a closed surface using the divergence theorem. 1½

(G) Test the convergence of $\int_{1}^{\infty} \frac{x\,dx}{2x^2 + 3x + 5}$, by comparison test. 1½

(H) Prove that $\beta(m, n) = \beta(n, m)$. 1½